Dr. Marques Sophie
Office 519

Number theory

Fall Semester 2013

marques@cims.nyu.edu

MIDTERM

Maximal Score: 200 points

Wednesday october 16th

Problem 1: ($\star$) 50 points

1. Give all the congruence classes mod 99 solutions of the following system of congruences:
$$\begin{cases} 2x & \equiv & 1 \; mod \; 9 \\ 5x & \equiv & 3 \; mod 11 \end{cases}$$

2. Decide whenever the following linear diophantine equations have any solutions. If so give the general solutions, if not says why there is no solutions

   (a) $6x + 51y = 22$;

   (b) $162x + 64y = 4$.

Problem 2: ($\star\star$) 40 points (The following questions are independents.)

1. Prove that the fraction $\frac{21n+4}{14n+3}$ is irreducible, for any $n \in \mathbb{Z}$.
2. Prove that $\frac{n^3+2n}{3}$ is an integer, for any $n \in \mathbb{Z}$.

Problem 3: ($\star$) 10 points
What is the remainder of 5^{2012} when divided by 11?

Problem 4: ($\star$) 15 points
Suppose that a, b and c are positive integers with $b|c$ and $m = [a, c]$. Prove that $[a, b] \leq m$.

Problem 5: ($\star$) 50 points
Use induction to show that

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

Exercise 6: ($\star\star$) 35 points

1. Prove that if $a \equiv b \ mod \ p$, for every prime p, then $a = b$.
2. Let $a, m, n, b, c \in \mathbb{N}$ such that $(m, n) = 1$. Suppose that

$$\begin{cases} a^b \equiv 1 \ mod \ m \\ a^c \equiv 1 \ mod \ n \end{cases}$$

   Prove that

$$a^{bc} \equiv \ 1 \ mod \ mn$$